SELF-SIMILAR MOTION OF A GAS HEATED BY A NONEQUILIBRIUM CONTINUOUS RADIATION SPECTRUM

V. M. Krol and I. V. Nemchinov

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 5, pp. 32–37, 1968

In this paper we discuss the motion of the vapor formed during the evaporation of a solid by a continuous radiation spectrum. The vapor is assumed to be heated by this radiation to a temperature T much higher than the phase-transition temperature T_v and much higher than the temperature T_i at which significant ionization of the vapor begins.

In the case, T_v and T_i can be neglected (as can the heat of evaporation Q_v and the energy Q_i expended on ionization). As a result of this motion, the vapor has a density ρ much lower than the density ρ_0 of the solid. It can therefore be assumed that the heating wave moves through an absolutely cold and infinitely dense gas. At the same time, the vapor temperature is assumed low enough that reradiation can be neglected. The radiation-absorption coefficient $\varkappa$ for the ionized vapor can be described by a power-law dependence on T and ρ for certain ranges of T, ε, and the photon energy ε. In this case, the motion of the gas is a self-similar problem. The spectrum and angular distribution of the incident radiation $[\varphi(\varepsilon,\theta)]$ and the $\varkappa$ and ε dependences can be arbitrary. A system of ordinary differential equations is found and solved.

Intense radiation incident on a solid surface will evaporate the solid. If the absorption coefficient $\varkappa$ of the vapor and the flux density q of the radiation are high enough, the escaping vapor will be heated to a high temperature in a relatively short time. This temperature will not only be much higher than the evaporation temperature T_v, but it will also be higher than the "ionization temperature" T_i. If the internal energy per unit mass of the vapor is much higher than the heat of evaporation Q_v and the energy Q_i expended on ionization, and if the vapor density ρ is much lower than the initial density ρ_0 as a result of its escape, then the problem of the motion and heating of the vapor can be simplified through the assumptions

$$T_v = T_i = Q_v = Q_i = 0, \quad \rho_0 = \infty \quad (v_0 = 1 / \rho_0 = 0) \qquad (0.1)$$

(here and below, v is the specific volume). We can therefore assume that the heating wave moves through an infinitely dense and absolutely cold gas. In the region of multiple and complete ionization, the ionized-vapor absorption coefficient $\varkappa$, associated with free-free electron transitions in the field of ions, and bound-free transitions from the higher-lying states of atoms and ions, has an approximately power-law dependence on T and ρ [1], or on p and ρ (p is the pressure):

$$\varkappa = k\varphi(\varepsilon)\, T^{-\alpha}\rho^{\beta} = K\varphi(\varepsilon)\, p^{b}\rho^{-a}. \qquad (0.2)$$

Here k and K are numerical coefficients which depend on the substance and on the ranges of T, ρ, and ε in which (0.2) is used. For a completely ionized gas, we have $\alpha = 3/2$, $\beta = 1$, $a = -5/2$, $b = -3/2$, and $\varphi(\varepsilon) = \varepsilon^2$ when $\varepsilon \ll T$; or $\alpha = 3/2$, $\beta = 1$, $a = 3/2$, and $b = -1/2$ when when $\varepsilon \gg T$. We assume that (0.2) holds for any T, for approximation (0.1). We assume the ratio of specific heats γ to be constant for a certain temperature range in the range of multiple and complete ionization. With these simplifying approximations, the problem of the planar, transient flow of a gas heated by a beam of monochromatic radiation is a self-similar problem. It has been studied in [2,3]. It is shown below that the analogous problem of the motion of a gas heated by a nonequilibrium continuous radiation spectrum is also self-similar.

For a partially ionized gas, approximation (0.2) is usualy satisfied only for the long-wavelength part of the incident spectrum. For the short-wavelength part of the spectrum (that is, for photons whose energy is close to or greater than the ionization potential characteristic of the ions for the given temperature range, and which are capable of direct photoionization of these ions from the ground or first excited status), the absorption coefficient is usually much smaller (by several orders of magnitude). This "hard" radiation penetrates a short distance into the solid, causing intense heating of a thin surface layer of small mass. An afterionization wave propagates through the substance, moving under the influence of the radiation flux in the hard part of the spectrum; if the temperature of the surface layer is close to the source temperature T_e, and reradiation becomes important, there will also be a thermal wave [1]. Since the energy expended in heating is large in these waves, their propagation velocity is small (in comparison with that of the wave of evaporation, initial ionization, and heating of the plasma by the long-wavelength part of the spectrum), even if the hard and soft parts of the incidence spectrum have comparable energies (E_h and E_s). Also, the intense reradiation by the thermal wave in the hard part of the spectrum increases its propagation velocity. Finally, the energy in the short-wavelength part of the spectrum may in general be small because of self-adsorption in the source itself (for example, adsorption of the short-wavelength radiation in the cold working gas ahead of a shock wave front in an explosive source [4]). Accordingly, the heating waves for the various parts of the source spectrum may propagate differently. Since the mass of the surface layer heated by the short-wavelength part of the spectrum is small, the pressure produced as a result of of the disintegration of the surface layer is small when E_h is of the order order of E_s or, especially, when $E_h \ll E_s$; that is, the hydrodynamic effects of the heating and surface-layer disintegration on the motion and and heating of the deep layers heated by the "basic" part of the spectrum can also be neglected. The high temperature and low density of this layer only facilitate the penetration of the long-wavelength part of the spectrum into the deeper layers; however, because of the small mass of this layer, even this phenomenon has little effect on the hydrodynamic processes in the deeper layers. Accordingly, Eq. (0.2) can frequently be assumed valid for the basic part of the spectrum in the case of a partially ionized gas, also; the rest of the spectrum may simply be neglected. These restrictions on the applicability of the self-similar problem are generally removed in the case of a completely ionized gas. A state close to that of complete ionization arises when two ionization potentials typical of a given temperature range are greatly different (this occurs, for example, in the case of the alkaline metals, and also when one atomic shell has been essentially ionized, while another has not yet started to be ionized; e.g., the L- and K-shells or the M- and L-shells).

We consider here the case in which the heating is caused by nonequilibrium radiation, that is, radiation such that the intrinisic radiation of the vapor may be neglected. This is a valid assumption when the vapor temperature is considerably below the source temperature T_e, or, more accurately, when the following condition holds (for a Planckian source spectrum):

$$W\sigma T_e^4 \chi\left(\frac{\varepsilon_1}{T_e}, \frac{\varepsilon_2}{T_e}\right) \gg \sigma T^4 \chi\left(\frac{\varepsilon_1}{T}, \frac{\varepsilon_2}{T}\right). \qquad (0.3)$$

Here W is the source-radiation dilution coefficient due to geometric factors, σ is the Stefan-Boltzmann constant, ε_1 and ε_2 are the boundaries of the "basic part" of the spectrum, and χ is the fraction of the spectral energy of a Planckian source with a temperature T_e or T for photons with energies $\varepsilon_1 \le \varepsilon \le \varepsilon_2$. We note that the boundaries ε_1 and ε_2 for the source and vapor-radiation spectra are sometimes slightly different, but condition (0.3) can be easily modified for this situation or for a non-Planckian source spectrum.

For our problem, the radiation intensity $J = J(m, t, \varepsilon, \theta)$ is a function of four variables: the time t, the Lagrangian mass coordinate m, the photon energy ε, and the angle θ between the direction of motion and the beam direction. The intensity $J_0 = J(0, t, \varepsilon, \theta)$ of the radiation incident on the boundary m = 0 is assumed to be a given function. In the self-similar problem, J can be represented as

$$J = t^{\lambda} J(mt^{-n}, \varepsilon, \theta). \tag{0.4}$$

This can be done (when conditions (0.1)-(0.3) are satisfied) when J_0 can be represented by

$$J_0 = t^{\lambda}\psi(\varepsilon, \theta) \quad (\varepsilon_1 \leqslant \varepsilon \leqslant \varepsilon_2, \ \theta_1 \leqslant \theta \leqslant \theta_2). \tag{0.5}$$

If the source spectrum is Planckian, condition (0.5) requires that T_e = const. In this case, the power-law time dependence of the intensity J_0 may reflect, for example, motion of the radiation source toward the irradiated surface; in this case, however, the limiting angle θ_2 of the incident radiation also changes (usually, $\theta_1 = 0$). As before, the problem is self-similar if these angles $\theta_2(t)$ are always small; that is, if the radiation is almost completely unidirectional. The arbitrary nature of the function $\psi(\varepsilon, \theta)$, which shows the spectrum and angular distribution of the source radiation, and the arbitrary nature of the function $\varphi(\varepsilon)$, which shows the dependence of the absorption coefficient on the photon energy, permit us to analyze the effects of these functions on the heating and motion of the substance for the case of the self-similar solution.

1. The equations of motion, continuity, energy, and transport of the source radiation are

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0, \qquad \frac{\partial v}{\partial t} = \frac{\partial u}{\partial m}, \tag{1.1}$$

$$\frac{\partial e}{\partial t} + p\frac{\partial v}{\partial t} + \frac{\partial q}{\partial m} = 0, \qquad e = \frac{pv}{\gamma - 1}, \tag{1.2}$$

$$q = \int_{\varepsilon_1}^{\varepsilon_2} q_\varepsilon \, d\varepsilon, \quad q_\varepsilon = 2\int_{\theta_1}^{\theta_2} J \sin\theta\cos\theta \, d\theta,$$

$$\cos\theta \frac{\partial J}{\partial m} = -\varkappa J. \tag{1.3}$$

When γ = const, and when condition (0.2) holds the energy and radiation-transport equations can be written

$$v\frac{\partial p}{\partial t} + \gamma p \frac{\partial v}{\partial t} = 2(\gamma - 1)\int_{\varepsilon_1}^{\varepsilon_2}\int_{\theta_1}^{\theta_2} J \sin\theta \, d\theta \, d\varepsilon,$$

$$\cos\theta \frac{\partial J}{\partial m} = -kp^b v^a \varphi(\varepsilon) J. \tag{1.4}$$

At initial time, the gas which occupies a half-space, is assumed cold and motionless:

$$u(m, 0) = p(m, 0) = 0, \qquad v(m, 0) = v_0. \tag{1.5}$$

The boundary conditions for the problem are governed by J_0 and (in general) by the pressure p(0,t) of a piston or by its velocity u(0,t). For the self-similar problem, the piston must, of course, move according to a special power law. However, the case of greatest interest is usually disintegration in vacuum: p(0,t) = 0.

We introduce the self-similar variables V, P, U, Q, and x:

$$v(m, t) = t^{\omega_v} q_0^{\lambda_v} k^{-3/c} V(x),$$

$$p(m, t) = t^{\omega_p} q_0^{\lambda_p} k^{1/c} P(x),$$

$$u(m, t) = t^{\omega_u} q_0^{\lambda_u} k^{-1/c} U(x),$$

$$J(m, t, \varepsilon, \theta) = t^{\alpha} q_0 Q(x, \varepsilon, \theta),$$

$$x = mt^{-n} q_0^{-(a+b)/c} k^{-2/c},$$

$$\begin{gathered} c = 3a - b - 2, \qquad n = [\alpha(a + b) + 3a - b]/c, \\ c\omega_v = -3 + \alpha(2b + 1), \qquad c\omega_p = 1 + \alpha(2a - 1), \\ c\omega_u = \alpha(a - b - 1) - 1, \qquad c\lambda_v = -(2b + 1), \\ c\lambda_p = 2a - 1, \qquad c\lambda_u = a - b - 1. \end{gathered} \tag{1.6}$$

With $\alpha = 0$, $a = -3/2$, and $b = -1/2$, we have $c = -6$, $n = 4/3$, $\omega_v = 1/2$, $\omega_p = -1/6$, $\omega_u = 1/6$, $\lambda_v = 0$, $\lambda_p = 2/3$, and $\lambda_u = 1/3$.

Substituting (1.6) into (1.1) and (1.4), we find a system of self-similar equations:

$$\begin{gathered} rx(VP' + \gamma PV') + \\ + [1 - 3\gamma + \alpha(2a - 1 - \gamma(2b + 1))/c]PV = \\ = (\gamma - 1)\left(\int_{\theta_1}^{\theta_2}\int_{\varepsilon_1}^{\varepsilon_2} Q(x, \varepsilon, \theta)\sin\theta \, d\theta \, d\varepsilon\right), \\ P' + rxU' + [(\alpha(a - b - 1) - 1)/c]U = 0, \\ rxV' - [3 + \alpha(2b + 1)/c]V = U' \\ \cos\theta\frac{\partial Q(x, \varepsilon, \theta)}{\partial x} = -\varphi(\varepsilon)V^a P^b Q(x, \varepsilon, \theta), \\ cr = b - 3a - \alpha(a + b). \end{gathered} \tag{1.7}$$

We note that in the case of a finite initial density ρ_0, the problem is self-similar only when $\alpha = -3/(1 + 2b)$; however, in the limiting case of infinite density ($v_0 = 1/\rho_0 = 0$), the value of α is arbitrary (the limiting transition for the case of monochromatic radiation was analyzed in [3]).

For a self-similar problem, the piston should move according to

$$u(0, t) = u_0 t^{\omega_u} \quad (\text{or } p(0, t) = p_0 t^{\omega_p}). \tag{1.8}$$

Because of the substitution of variables which has been made, the initial and boundary conditions become boundary conditions for Eqs. (1.7):

$$\begin{gathered} U = U_0 \ (\text{or } P = P_0), \quad Q = \psi(\varepsilon, \theta) \quad \text{for } (x = 0), \\ U = P = 0, \qquad V = 1 \quad \text{for } x = \infty. \end{gathered} \tag{1.9}$$

2. The radiation-transport equation is actually a system of an infinite number of equations, since ε and θ take on an infinite set of values on the intervals $(\varepsilon_1, \varepsilon_2)$ and (θ_1, θ_2). In practice, however, only one of these equations need be solved, for the radiation intensity of photons of energy ε_0 progagating at an angle θ_0; then the other intensities can be calculated. The solution for the radiation-transport equation can be written in the form

$$J = J_0 \exp(-\tau) = J_0 \exp\left(-\int_0^m \varkappa \, dm\right) \quad (t = \text{const}). \tag{2.1}$$

Here τ is the optical thickness of the layer for photons of energy ε propagating at an angle θ. If condition (0.2) is satisfied, we have

$$\tau(m, t, \varepsilon, \theta) = \frac{\varphi(\varepsilon)\cos\theta_0}{\varphi(\varepsilon)\cos\theta}\tau(m, t, \varepsilon_0, \theta_0). \tag{2.2}$$

Accordingly, we have

$$J(m, t, \varepsilon, \theta) = J(0, t, \varepsilon, \theta)\left[\frac{J(m, t, \varepsilon_0, \theta_0)}{J(0, t, \varepsilon_0, \theta_0)}\right]^{\psi},$$
$$\psi = \frac{\varphi(\varepsilon)\cos\theta_0}{\varphi(\varepsilon_0)\cos\theta}. \tag{2.3}$$

Transforming to self-similar variables, we find

$$Q(x, \varepsilon, \theta) = Q(0, \varepsilon, \theta)\left[\frac{Q(x, \varepsilon_0, \theta_0)}{Q(0, \varepsilon_0, \theta_0)}\right]^{\psi}. \tag{2.4}$$

This property considerably simplifies the solution of system (1.7).

3. It is not difficult to see that, as in the case of monochromatic radiation (see [3]), the number of boundary conditions (1.9) for system (1.7) is one greater than the number of equations. It follows from the discussion in [3] that the solution of this problem is also discontinuous. The proof in [3] that no more than one discontinuity may occur in the desired solution remains valid here, provided that this solution does not pass through any internal singularity (in [3], it was argued that such solutions do not occur); also, the results found in an analysis of the behavior of the solution with variations in the initial density ρ_0 remain valid here.

If the incident flux is bounded in energy, and ρ_0 is finite, there exists a finite point x_1 at which

$$U(x_1) = P(x_1) = Q(\varepsilon_0, \theta_0, x_1) = 0. \tag{3.1}$$

According to (2.4), all the other values of Q vanish. This point is singular because of an indeterminate form of the 0/0 type in the radiation-transport equation ($\varkappa \to \infty$ as $p \to 0$ and $J \to 0$). For large optical thicknesses, the dominant term in the energy equation is that of the intensity with the greatest photon energy ε_2 (for $\varphi(\varepsilon)$ which decreases with ε) and the least angle θ_1 (beams at oblique incidence are absorbed more intensely). This follows from (2.3) or (2.4). In a sufficiently small neighborhood of the point x_1, the solution is, within terms of a higher order of smallness (for $b < 0$),

$$P\,[-b\,V_1^a\,(x_1 - x)]^{-1/b}, \quad V = V_1 - P\,/\,r^2x_1^2,$$
$$U = P\,/\,rx_1, \quad 2\,(\gamma - 1)\Delta\varepsilon\,\Delta\,\theta Q\,(\varepsilon_2, \theta_1) = rx_1V_1P. \tag{3.2}$$

On the intervals $\varepsilon_2 \leq \varepsilon \leq \varepsilon_2 + \Delta\varepsilon$ and $\theta_1 \leq \theta \leq \theta_1 + \Delta\theta$, we may assume

$$J(\varepsilon, \theta) \approx J(\varepsilon_2, \theta_1).$$

The point $x = 0$ is also singular, but of the "cusp" type. The solution in the neighborhood of this point for $P_0 > 0$ and for $\alpha = -3/(1 + 2b)$ (the case of a finite initial density) is

$$V = V_0 + Ax^{(a-1)/b\gamma} +$$
$$+ [2P_0^{1+b}V_0^{1+a} + U_0V_0\,(2\,(b - 1)\,/\,k -$$
$$- r)]\,x\,/\,[2P_0\,(a - 1) - rk\gamma\,P_0],$$
$$U = U_0 - rx\,(V - V_0), \quad P = P_0 + U_0\,x\,/\,k,$$
$$Q\,(\varepsilon_0, \theta_0, x) =$$
$$= Q\,(\varepsilon_0, \theta_0, 0)\,[1 - V_0^a\,P_0^b\,x\varphi\,(\varepsilon_0)\,/\cos\theta_0]. \tag{3.3}$$

Here A is an arbitrary constant; $k = 1 + 2b$; and P_0, V_0, and $Q(\varepsilon, \theta, 0)$ are related by

$$\int_{\theta_1}^{\theta_2}\int_{\varepsilon_1}^{\varepsilon_2} Q\,(\varepsilon, \theta, 0)\sin\theta\,d\theta\,d\varepsilon = \frac{2V_0^{1-a}P_0^{1-b}}{k\,(1-\gamma)}. \tag{3.4}$$

For an infinite initial density ($v_0 = 0$), as in the case of monochromatic radiation, the corrdinate of the shock wave is $x_s = \infty$. The solution on the interval (x_1, ∞) is described by the constant functions

$$U = V = 0, \quad P = P_1. \tag{3.5}$$

The point x_1 is singular because of the "saddle-point" singularity which follows from the indeterminate form of the 0/0 type in the transport equation ($\varkappa \to \infty$ as $v \to 0$ for $a < 0$). The solution in its neighborhood ($x < x_1$) is

$$V = [a, \; P_1^b\,(x - x_1)]^{-1/a}, \quad P = P_1 - r^2x_1^2\,V,$$
$$U = rx_1\,V,$$
$$2\,(\gamma - 1)\,Q\,(\varepsilon_2, \theta_1)\,\Delta\varepsilon\Delta\theta = -\gamma rx_1P_1V, \tag{3.6}$$

The values of x_1 and P_1 are governed by the boundary conditions at $x = 0$.

The boundary-value problem is solved by an integration of the system of equations from $x = x_1$ to $x = 0$ and selection of x_1 and x_d (the discontinuity coordinate—the shock-wave front) or P_1 at $v_0 = 0$ in such a manner that the two following conditions hold at $x = 0$:

$$P\,(0) = P_0 \;(\text{or} \quad U\,(0) = U_0),$$
$$J\,(0, \varepsilon_0, \theta_0) = \psi\,(\varepsilon_0, \theta_0). \tag{3.7}$$

When the second of conditions (3.7) does not hold, the intensity of the incident radiation differs from the given intensity not only in total flux, but also in angular and frequency distribution. This circumstance distinguishes the continuous-spectrum problem from that of monochromatic radiation. A special algorithm has been developed for the solution of this problem. It is based on the monotonic contraction of a rectangle about the desired point in the x_1, P_1 plane. Certain qualitative results about the nature of the solution as a function of x_1 and P_1, found in [3] in an analysis of the monochromatic problem, were used. The accompanying table shows some results found from the solution of the problem of the heating and motion of an initially infinitely dense medium ($v_0 = 0$) by a radiation flux having a truncated Planckian spectrum,

$$Q\,(0, z, \theta) = \left(\frac{z}{z_2}\right)^3\frac{\exp(z_2) - 1}{\exp(z) - 1}, \quad z = \frac{\varepsilon}{T_s},$$
$$0 \leqslant \theta \leqslant {}^1\!/_2\pi, \quad \varphi(\varepsilon) = \varepsilon^2, \quad z_1 \leqslant z \leqslant z_2, \tag{3.8}$$

with an account of the angular distribution of the intensity for an isotropic initial distribution for several values of z_1, z_2, a, b, and γ. The dimensionless average energy $\langle z\rangle$ shown in the table is equal to

$$\langle z\rangle = \int_{z_1}^{z_2}\frac{z^4\,dz}{\exp(z) - 1}\left(\int_{z_1}^{z_2}\frac{z^3\,dz}{\exp(z) - 1}\right)^{-1}. \tag{3.9}$$

From a comparison of the effects of radiation pulses with the spectrum (3.8) and pulses of monochromatic

TABLE 1.

z_1	z_2	$\langle z \rangle$	z_a	P_1	x_1
$a=-5/2,\ b=-3/2,\ \gamma=5/3$					
0.5	3	2.03	1.16	0.732	0.606
0.5	4	2.54	1.43	1.19	0.784
0.5	5	2.94	1.52	1.78	1.00
0.5	6	3.25	1.59	2.64	1.27
1.0	6	3.34	1.79	2.56	1.26
$a=-1,\ b=0,\ \gamma=6/5$					
0.5	3	2.03	1.31	0.483	0.814
0.5	4	2.54	1.54	0.648	0.877
0.5	5	2.94	1.79	0.893	0.936
0.5	6	3.25	2.07	1.25	0.991

radiation [3] of equal total energies, and on the basis of similarity arguments, we find that the pressures are the same in the two cases if the dimensionless energy of the monochromatic-radiation photons is equal to the value z_a shown in the table.

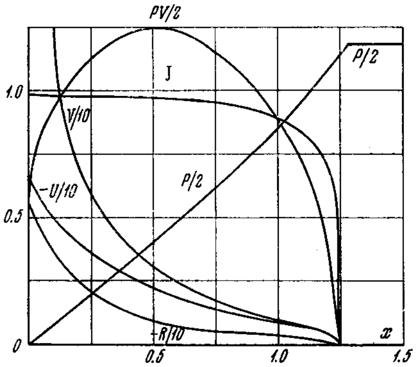

Fig. 1.

The accompanying figure shows self-similar profiles of V, P, U, and Q (x, z_2, θ) for this problem, for the case $z_1 = 1$, $z_2 = 6$, $a = -5/2$, $b = -3/2$, and $\gamma = 5/3$. According to Eq. (2.4), the dimensionless intensities Q (x, z, θ) have a power-law dependence on the intensity Q$(x, 6, 0)$ shown in the figure:

$$Q(x, z, \theta) = \frac{(^1/_6 z)^3 (e^6 - 1)}{e^z - 1} [Q(x, 6, 0)]^{\vartheta},$$

$$\vartheta = \left(\frac{6}{z}\right)^2 \frac{1}{\cos\theta}, \tag{3.10}$$

$$(1 \leqslant z \leqslant 6, \quad -{}^1/_2\pi \leqslant \theta \leqslant {}^1/_2\pi).$$

REFERENCES

1. Ya. B. Zel'dovich and Yu. P. Raizer, The Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian] 2-nd edition Nauka, Moscow, 1966.

2. Yu. V. Afanas'ev, V. M. Krol, O. N. Krokhin, and I. V. Nemchinov, "Gasdynamic processes during the heating of a substance by laser radiation," PMM, 30, no. 6, 1966.

3. V. M. Krol, "Planar self-similar motion of a heat-conducting gas heated by radiation," PMTF [Journal of Applied Mechanics and Technical Physics], no. 4, 1968.

4. I. F. Zharikov, I. V. Nemchinov, and M. A. Tsikulin, "Effects on a solid of the electromagnetic radiation produced by an explosive source," PMTF [Journal of Applied Mechanics and Technical Physics], no. 1, 1967.

23 May 1968 Moscow